Surgery Presentations for Dihedral Covering Links

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Setting

Let M be a compact connected oriented 3–manifold.
We can present M:

1. As an n–fold dihedral covering space of S^3 branched over $K \subset S^3$ with monodromy given by $\rho\colon\ \pi_1\left(\overline{S^3 - N(K)}\right) \twoheadrightarrow D_{2n}$.

   Good for classical topology.

2. Via surgery on a framed link $L \subset S^3$.

   Good for quantum topology.

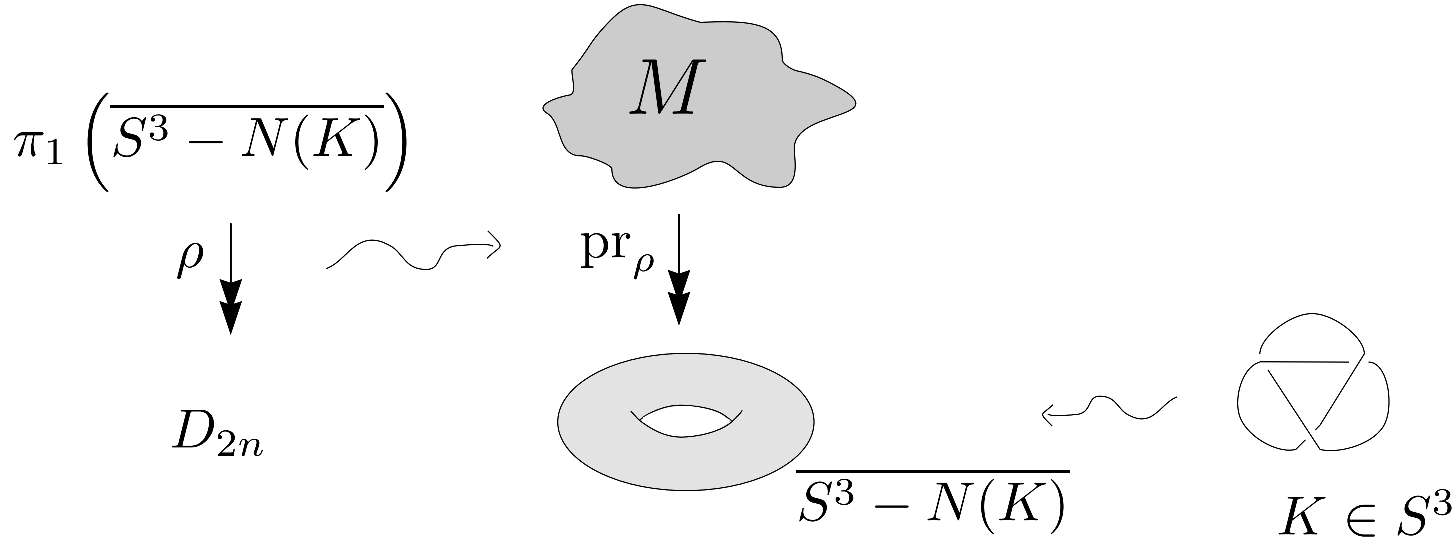
Motivations

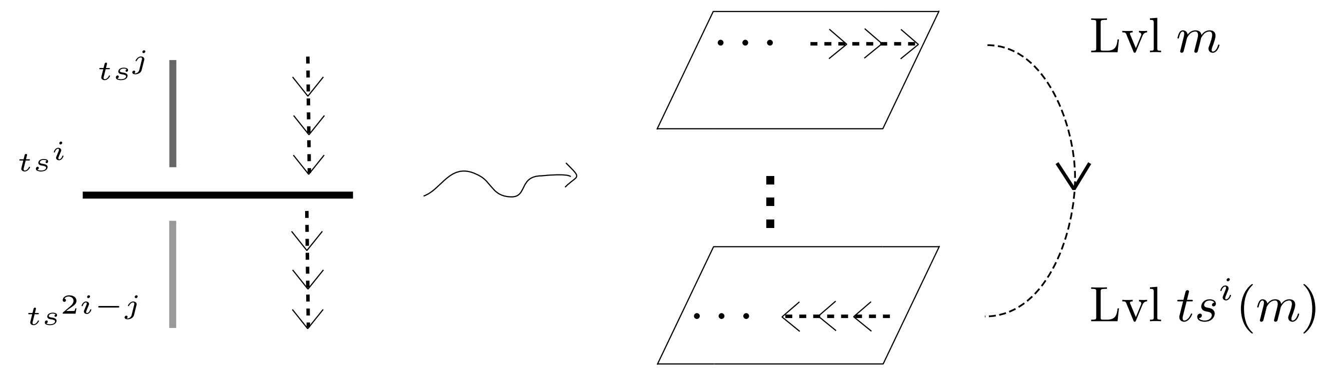
Why quantum topology for dihedral covering spaces?

- Non-commutative Alexander polynomial, 2–loop polynomial, rational Kontsevich invariant,...
- Apply quantum topology to covering linkage invariants, knot concordance, Casson–Walker–Lescop invariant,...

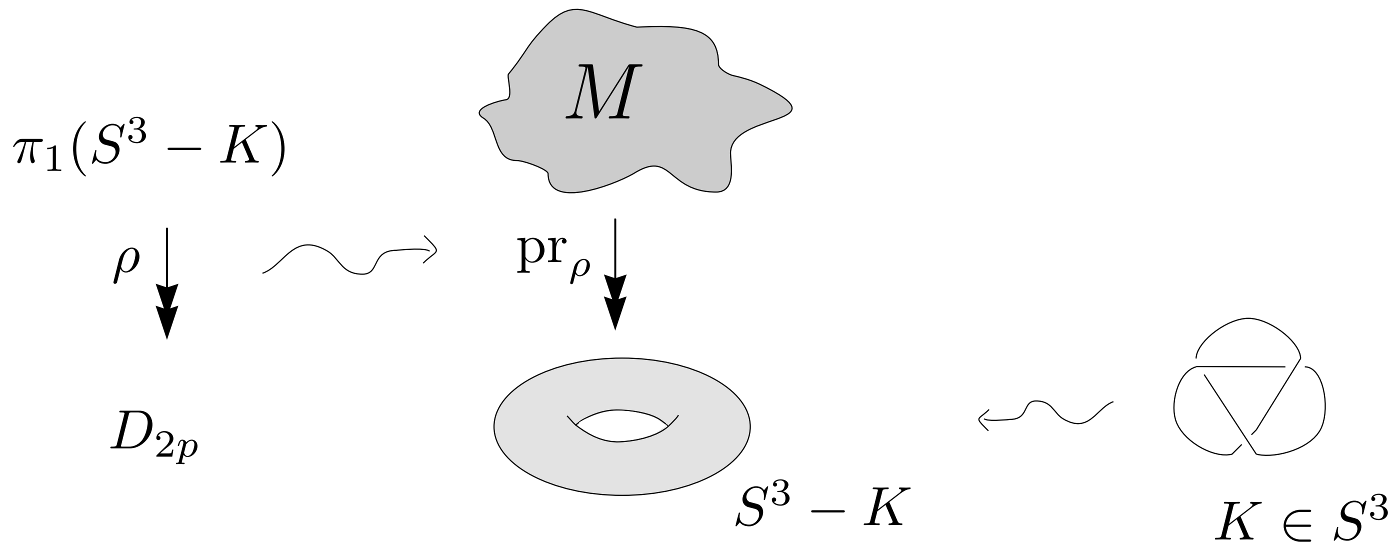
$$\begin{array}{ccccc}
\tilde{K} \subset M & \xleftarrow{\operatorname{surg}(\tilde{L})} & S^3 & \supset \tilde{L} \cup \tilde{\mathcal{O}} \\
\Big\downarrow{\scriptstyle \operatorname{pr}_\rho} & & \Big\downarrow{\scriptstyle \operatorname{pr}_{\rho'}} & \\
K \subset S^3 & \xleftarrow[\operatorname{surg}(L)]{} & S^3 & \supset L \cup \mathcal{O}
\end{array}$$

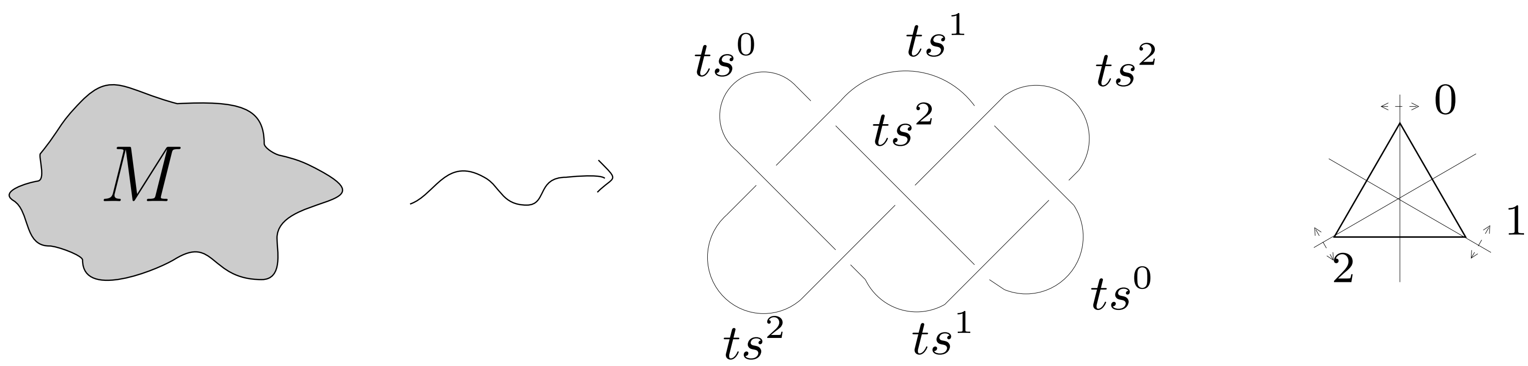
Dihedral Covering Presentation



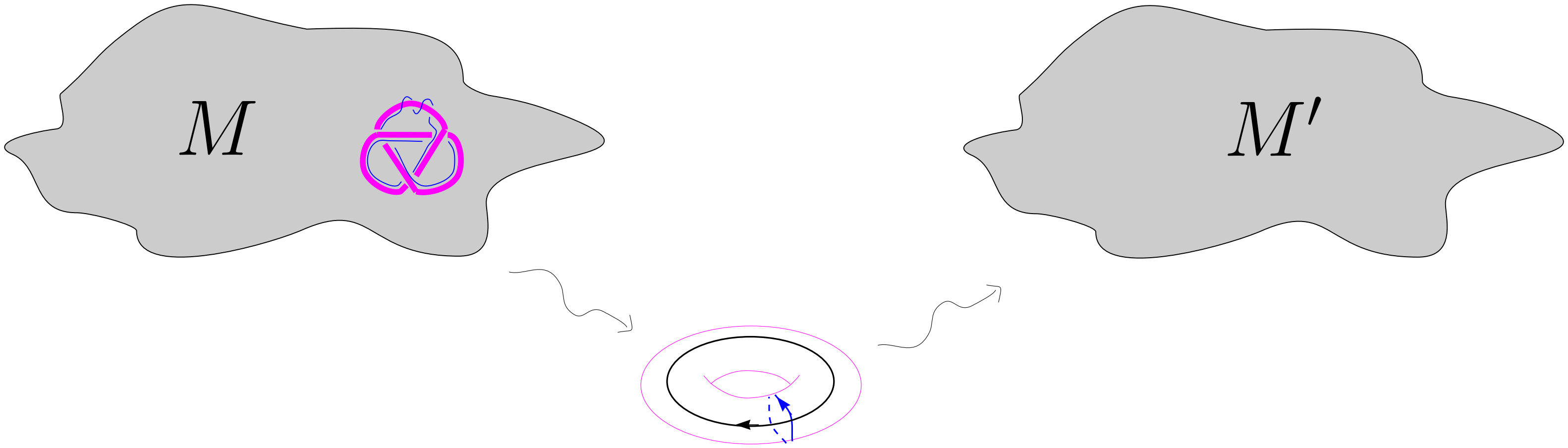


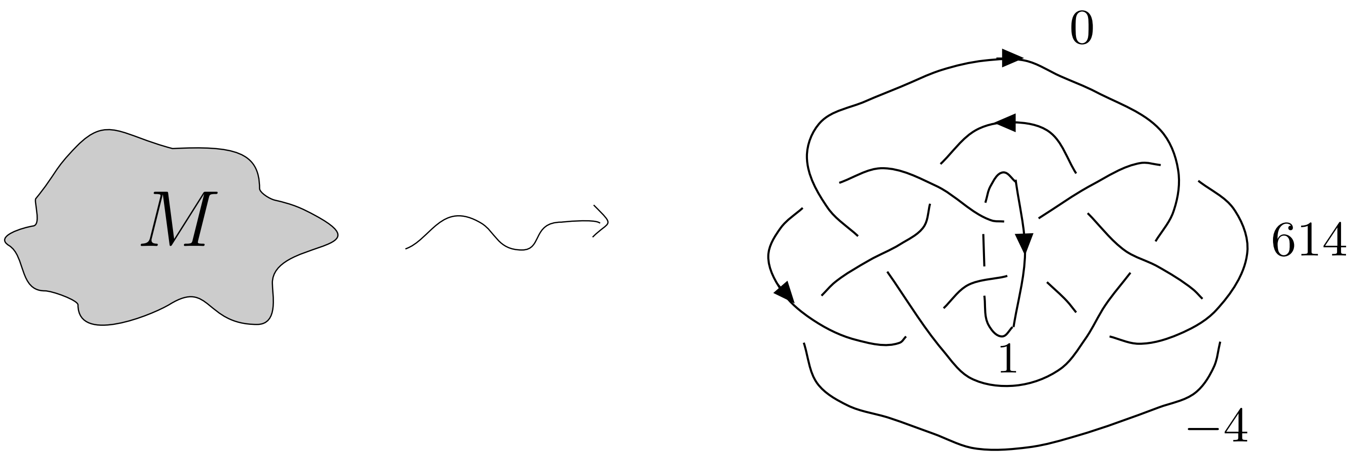
Dihedral Covering Presentation

- 

- 

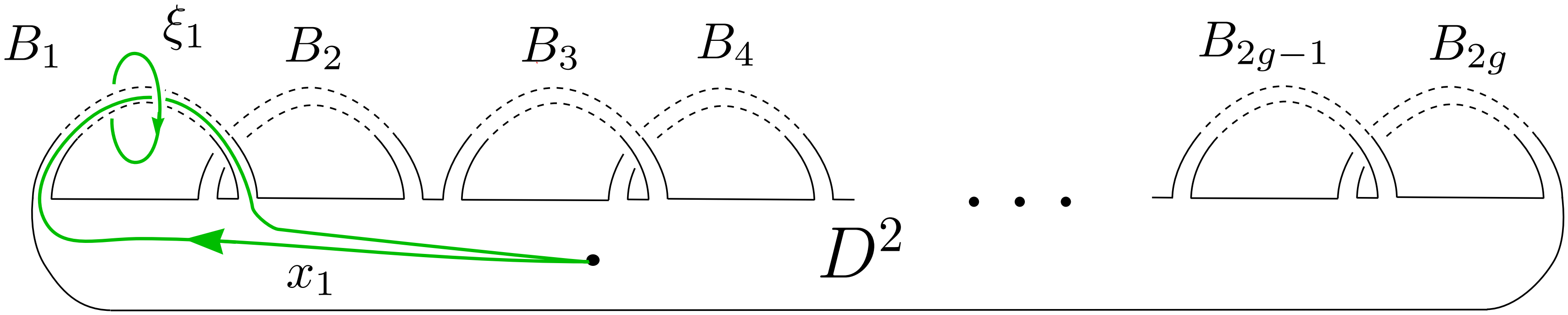
Surgery Presentation

- 

- 

Algebraic approach

- Choose bases for homology:



- Surgery in $\ker\rho$ and S-equivalence induce an equivalence relation on the Seifert matrix S and $\vec{v} := \big(\rho(\xi_1)\ldots,\rho(\xi_{2g})\big)$.

Algebraic approach

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- Modulo this equivalence relation:

$$(S,\vec{v}) \sim \left(\begin{pmatrix} (kn+\frac{n+1}{2}) & 0 \\ 1 & \frac{1-n}{2} \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right)$$

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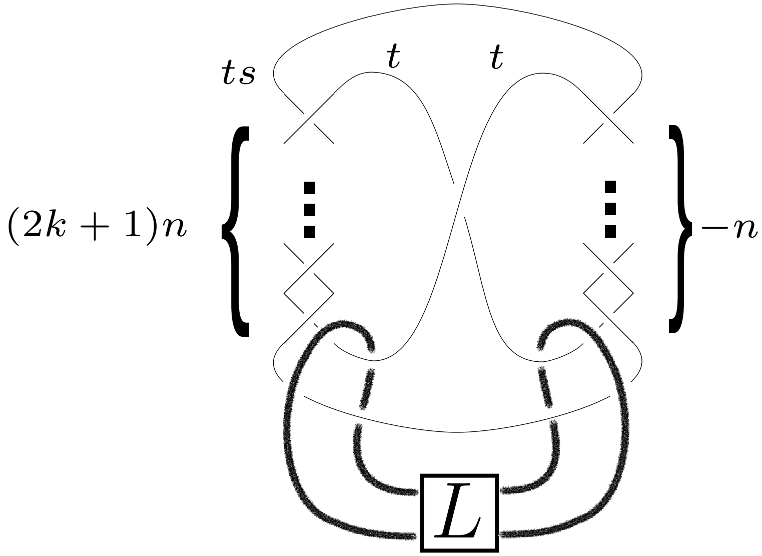
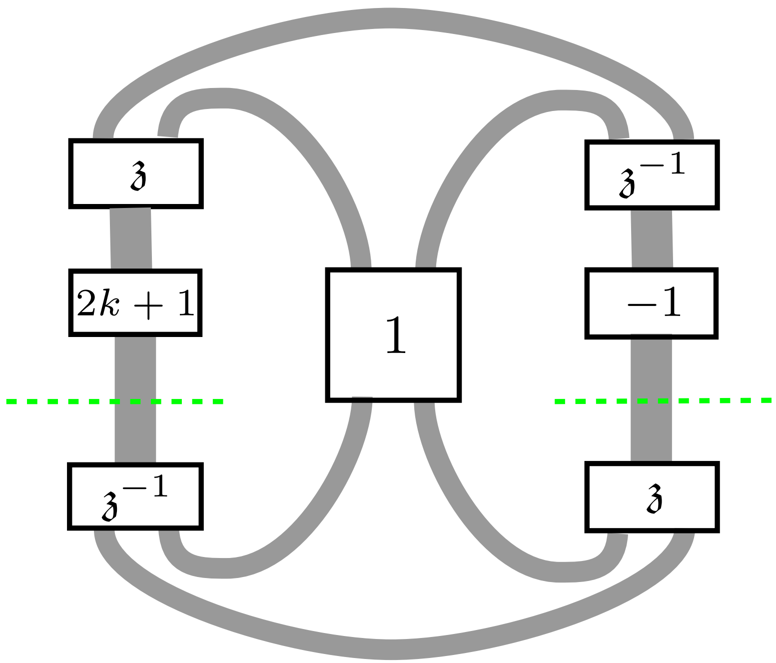
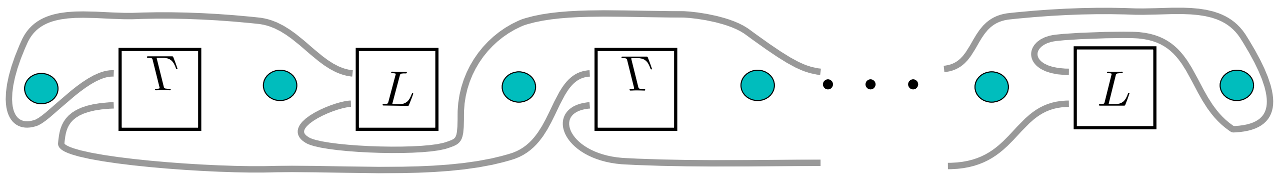
$$(K,\rho) \iff$$



Algebraic approach

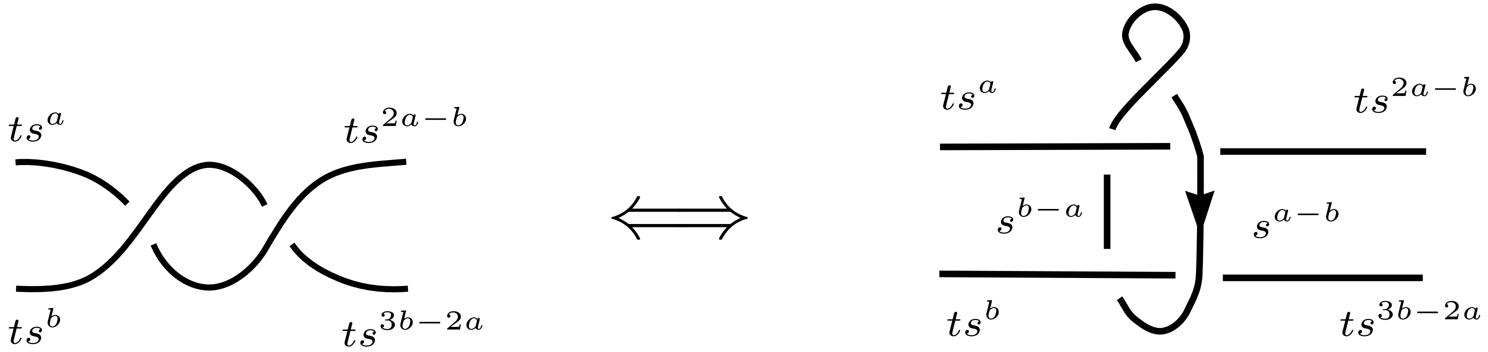
- Lift to the covering space:



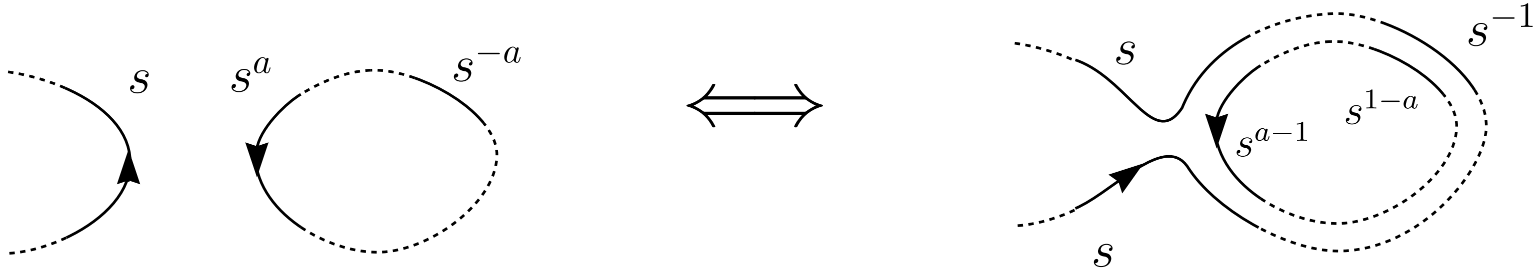


Untying approach

- Untie the knot by surgery.

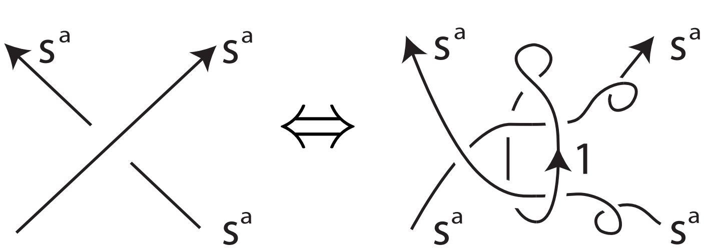


- Slide to get a distinguished component.

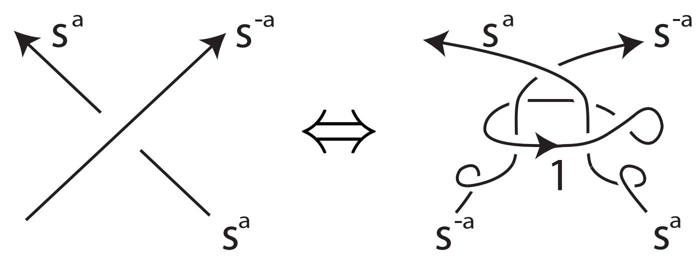


Untying approach

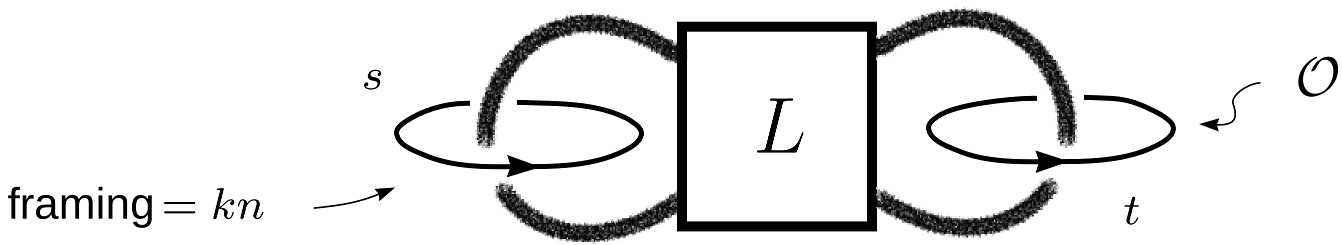
- Unlink the distinguished component from the unknot.



and

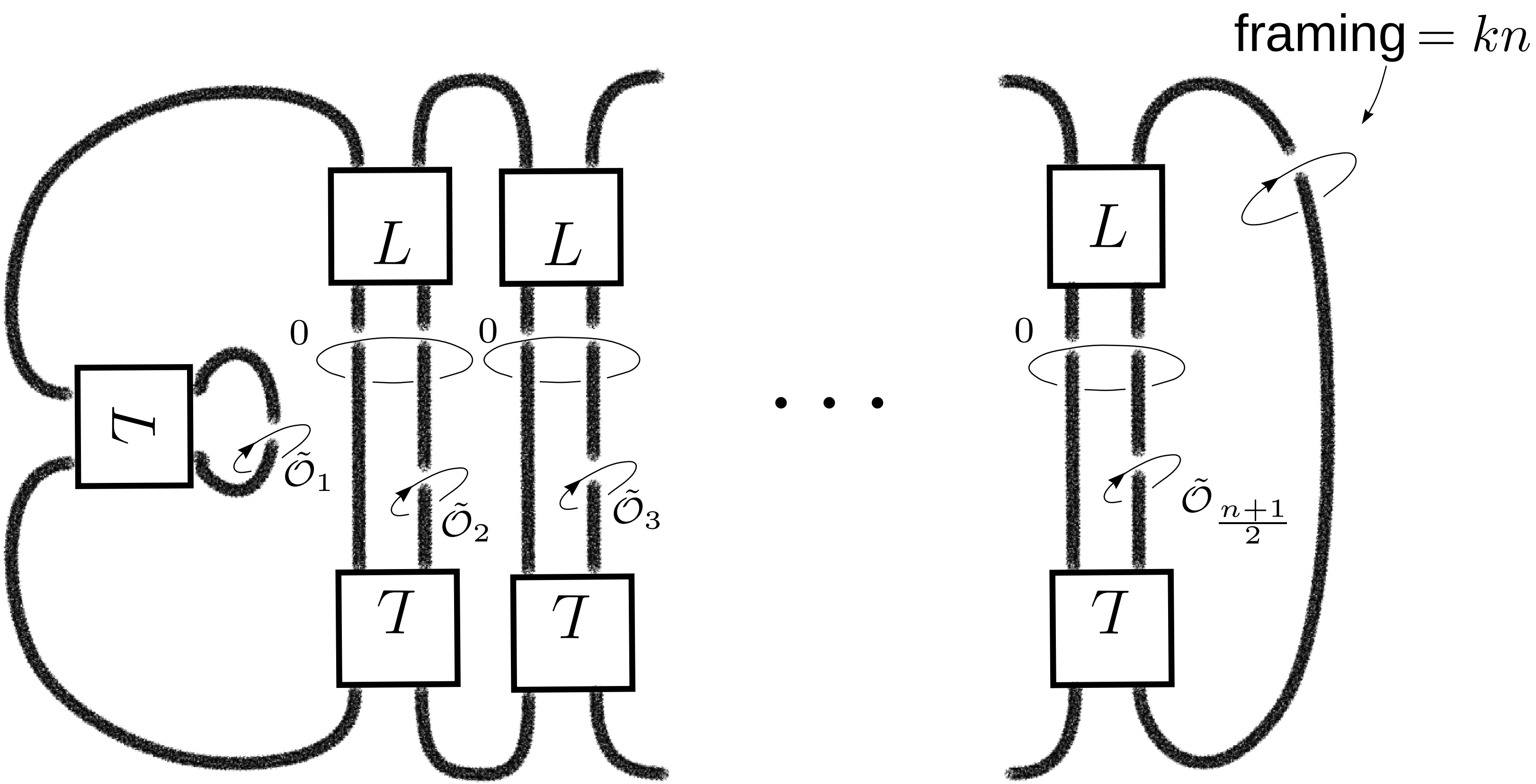


- Get a separated dihedral surgery presentation:

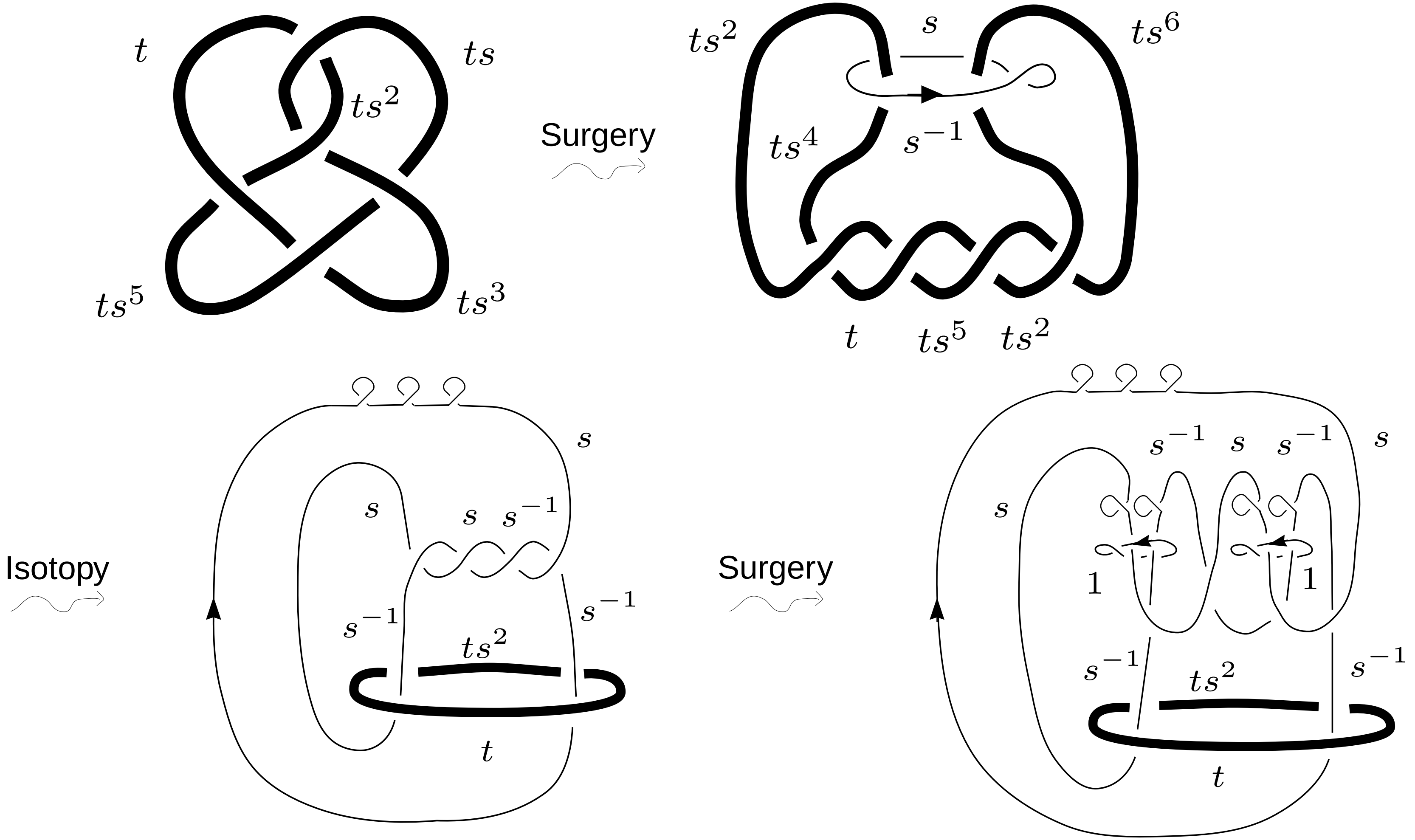


Untying approach

- Lift to the covering space:



Example



t
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ts^5
ts^3
Surgery
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ts^5
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Isotopy
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s^{-1}
s^{-1}
s^{-1}
ts^2
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Surgery
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s
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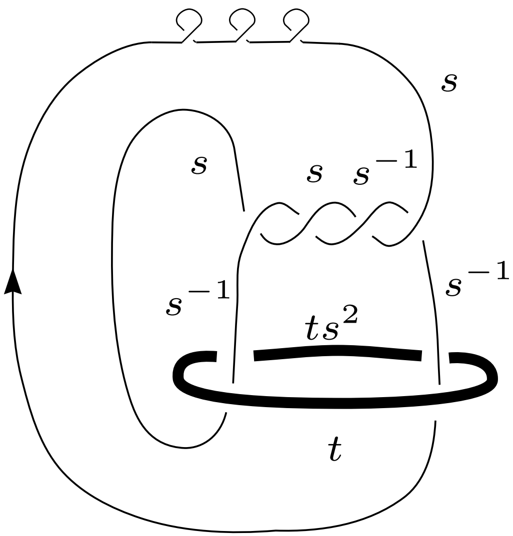
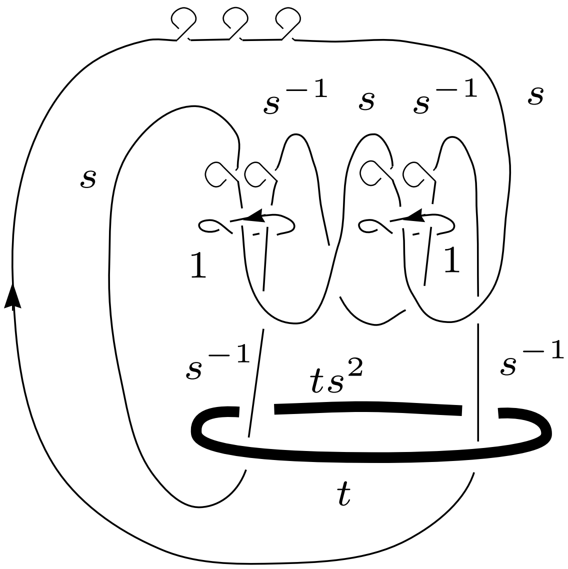
1
s^{-1}
ts^2
s^{-1}
t

Example

Isotopy

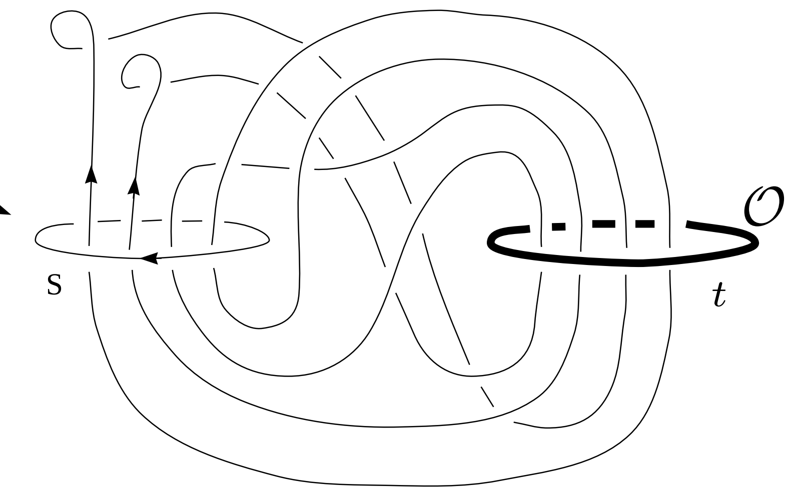


Surgery

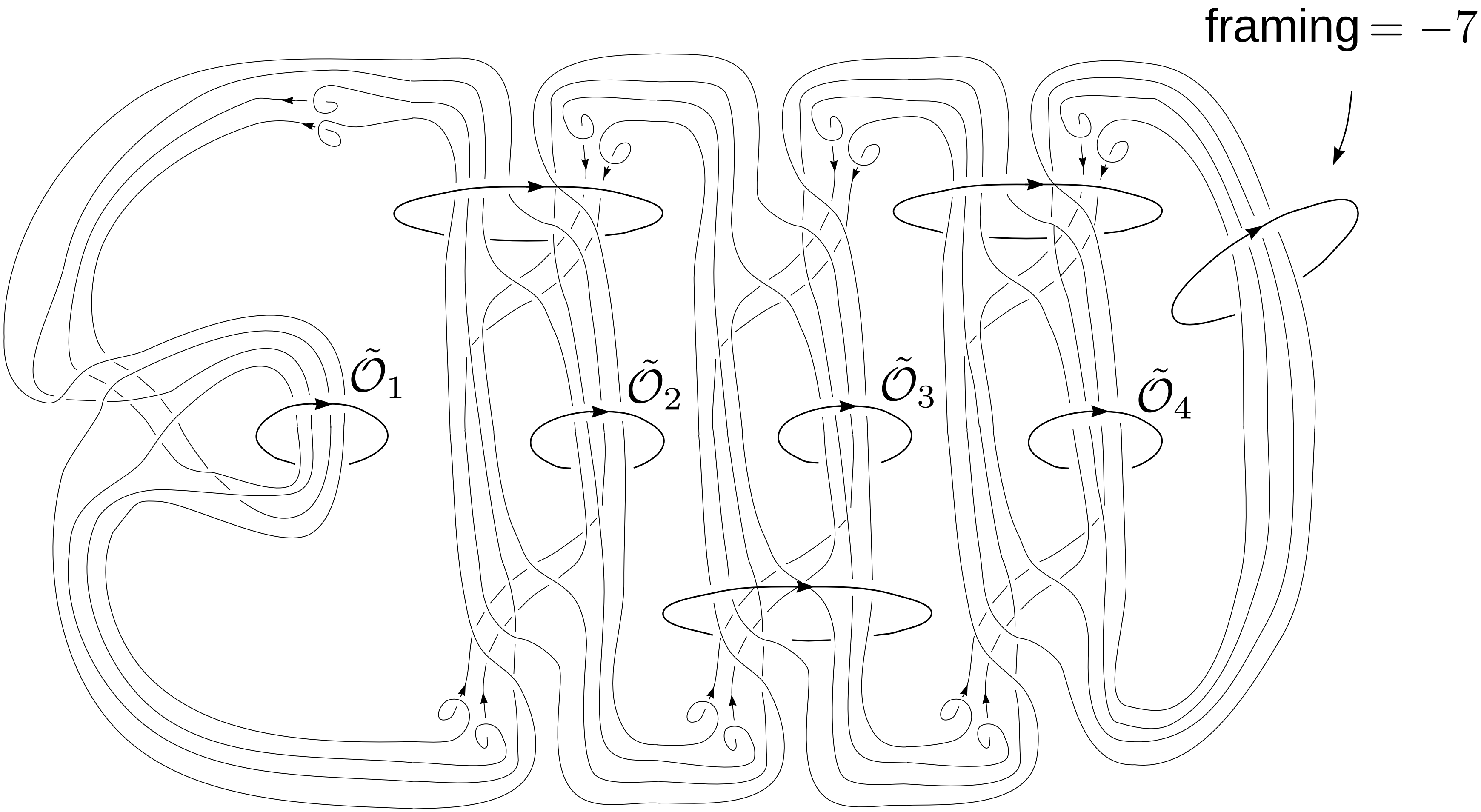


Isotopy

framing $= -7$



Example



framing = −7
$\tilde{\mathcal{O}}_1$
$\tilde{\mathcal{O}}_2$
$\tilde{\mathcal{O}}_3$
$\tilde{\mathcal{O}}_4$