GEOMETRIC APPLICATIONS OF WASSERSTEIN DISTANCE

SZYMON M. WALCZAK
UNIVERSITY OF ŁÓDŹ, POLAND

Lecture 1
Wasserstein distance and optimal transportation

The lecture will be devoted to the Wassertein distance of Borel probability measures, which arises from the optimal transportation theory [4] [5]. A number of examples will illustrate the nature of this metric, which is defined on the space of all Borel probability measures. Weak-$\star$ Metrization Theorem will be presented.

Lecture 2
Curvature of metric measure spaces I

The second lecture will present the notion of the lower curvature bound [3] for a metric measure spaces. Some basic concepts of metric geometry (length spaces, geodesics, geodesic spaces) will be recalled. A kind of a generalization of Wasserstein distance, namely the Wasserstein-Gromov distance of metric measure spaces and the relative entropy of measures will be the main definitions that will be presented.

Lecture 3
Curvature of metric measure spaces II

This lecture will be the continuation of the previous one. The lower curvature bound for a metric measure space will be presented. The most interesting Theorem which will be studied is the one about stability under convergence of the curvature bounds for metric measure spaces.

Lecture 4
Applications to differential geometry and foliations

Some recent results on application of the Wasserstein distance to the foliations will be presented. More precisely, the notion of metric diffusion along foliation will be introduced, and some results on metric diffusion along compact foliations with non-empty bad set [1] [2] will be studied.

References

[1] R. Edwards & K. Millett & D. Sullivan, *Foliations with all leaves compact*, Topology 16 (1977), 13-32
[2] D.B.A. Epstein, *Foliations with all leaves compact*, Ann. Inst. Fourier Grenoble 26 (1976), 265-282
[3] K.-T. Sturm, *On the geometry of metric measure spaces. I & II*, Acta Math., 196 (2006), 65–177.
[4] C. Villani, *Optimal transport*, Springer, 2009.
[5] C. Villani, *Topics in Optimal transportation*, AMS, 2003.